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Structure of neutron-rich Λ hypernuclei

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Abstract. Properties of light neutron-rich Λ hypernuclei ($^{16}_{\Lambda}$C, $^{12}_{\Lambda}$Be, and $^{11}_{\Lambda}$Li) are calculated within the Skyrme-Hartree-Fock approach. Interplay between hypernuclear interaction features and properties of these hypernuclei is studied. Response of weakly bound neutron states to hyperon addition depends generally on core distortion by hyperon, and it is essentially different for the different states. This response is especially sensitive to details of the ΛN interaction for $1p_{1/2}$ states. Implications of the nuclear spin-orbit potential and nuclear incompressibility in the neutron-rich system properties are inferred. Dependence of the Λ binding energies in hypernuclei on Z at fixed A is discussed.

PACS. 21.80.+a Hypernuclei – 21.60.Jz Hartree-Fock and random-phase approximations – 21.30.Fe Forces in hadronic systems and effective interactions

1 Introduction

Hypernuclei with neutron excess relate to two modern fields of nuclear physics: hypernuclear studies and physics of nuclei far from the stability line.

Firstly, they are interesting as an example of systems with grossly extended spatial distributions, which form neutron halos in some cases. Embedding of Λ hyperon into a halo system can provide information on its response to a perturbation. Also hypernuclear species with unstable nuclear cores can be examined.

On the other hand, hypernuclei with a neutron excess may allow to test hypernuclear interactions at low nuclear densities, particularly, the role of 3-body ΛNN force or density-dependent ΛN force can be revealed. Also charge symmetry breaking ΛN interaction can be studied.

Prospects for production of Λ hypernuclei with neutron halos have been discussed recently by Majling [1], who has pointed out that typical neutron-rich hypernuclei can be produced in (K^-,π^+) reaction, such as $^{9}_{\Lambda}$He, $^{11}_{\Lambda}$Li, $^{12}_{\Lambda}$Be, $^{16}_{\Lambda}$C. For an earlier treatment of some light neutron-rich hypernuclei see [2].

The first experimental attempt to detect Λ hypernuclei in the reaction (K^-,π^+) by stopped kaons performed at KEK has been reported recently by Kubota *et al.* [3]. They obtained upper limits for rates per a kaon for several targets. Some perspectives of such kind of experiments are discussed also in the FINUDA project at DAΦNE [4].

In this paper we consider p shell Λ hypernuclei with neutron excess in the frame of the Skyrme-Hartree-Fock approach. This rather simple calculation scheme is known to be able to reproduce qualitatively gross features of neutron-rich nuclei, and it is widely used in physics of nuclei with neutron excess (e.g., [5–9]). Importantly, this method allows us to take into account the influence of hyperon not only on weakly bound valence neutron(s), but also on the inner nucleons, named the core. We show that distortion of the core by the hyperon (the so-called core polarization) appears to be of substantial importance for such systems. The core polarization is generally a rather small effect, which is very difficult for experimental study. The first measurement was performed very recently at KEK for $^{7}_{\Lambda}$Li [10]. However, the polarization may be significant in some particular cases (for another example of significance of the hypernuclear core polarization see [11]).

The Skyrme-Hartree-Fock approach does not provide quantitative description of halo neutron separation energies without additional fitting. Particularly, the well-known abnormal level ordering (lowering of $2s$ states with respect to $1p$ and $1d$ ones in neutron-rich systems) cannot be reproduced. Since we need in correct asymptotic of neutron wave functions, we adopt the renormalization of single-neutron potentials, suggested by Bertsch *et al.* [7], which provides reasonable predictions for the rms radii [7, 8]. The calculation scheme is briefly described, and properties of neutron-rich hypernuclei are discussed in Sec. 2. We consider mainly $^{12}_{\Lambda}$Be and $^{16}_{\Lambda}$C, and show that their properties depend on details of interactions between the valence neutron and the core as well as between Λ and the core. We also give some comments concerning in $^{11}_{\Lambda}$Li. Sec. 3 is devoted to dependence of the Λ binding energies in hypernuclei on Z at fixed A. Some concluding remarks and discussion of prospects are presented in Sec. 4. Preliminary, partial results of the present study were reported in [12].

2 Influence of Λ hyperon on weakly bound neutron states and the ΛN interaction

2.1 Choice of potentials

We use the Skyrme-Hartree-Fock approach, adopted for Λ hypernuclei by Rayet [13], employing the Skyrme parametrization of hyperon-nucleon interaction as follows

$$\begin{aligned} \mathbf{v}_{\Lambda N}(\mathbf{r}_\Lambda, \mathbf{r}_N) &= \lambda_0 \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) \\ &+ \frac{1}{2}\lambda_1[\mathbf{k}'^2 \delta(\mathbf{r}_\Lambda - \mathbf{r}_N) + \delta(\mathbf{r}_\Lambda - \mathbf{r}_N)\mathbf{k}^2] \\ &+ \lambda_2 \mathbf{k}' \delta(\mathbf{r}_\Lambda - \mathbf{r}_N)\mathbf{k}; \qquad (1) \\ \mathbf{v}_{\Lambda NN}(\mathbf{r}_\Lambda, \mathbf{r}_1, \mathbf{r}_2) &= \lambda_3 \delta(\mathbf{r}_\Lambda - \mathbf{r}_1)\delta(\mathbf{r}_\Lambda - \mathbf{r}_2). \end{aligned}$$

All notations are standard [13,14].

This method is able to reproduce fairly well the observed spectra of Λ hypernuclei from light to heavy ones. A lot of parameter sets for potential (1) was fitted to the spectra [15–18]. The following point is substantial for our study: different interactions providing Λ binding energies close to each other can lead to rather different polarization of host nuclei by the hyperon. In terms of the Skyrme potential, the polarizing property is mostly governed by the strength of three-body ΛNN or density-dependent ΛN forces; that is, a density independent two-body ΛN force induces the core contraction, and a great contribution of a ΛNN (or density dependent ΛN) force can result in a core dilatation at the same binding energy. For details, see references collected in [19]. The most but not all the potentials, fitted to experimental spectra, offer slight polarizing abilities, incorporating ΛNN forces of moderate strengths.

It should be noted that the Skyrme ΛNN force and the density-dependent ΛN act usually quite similarly [17, 20]. Potentials used are formulated with three-body contributions, but our conclusions are valid for the density-dependent ΛN forces as well.

In this work we present the results of calculations with three parameter sets, which produce different polarization of the host nuclei. Firstly, we use the 5th set from [15] (this set is denoted hereafter as YBZ5). It furnishes a good agreement [15,18] with experimental Λ binding energies. This set involves the ΛNN force of a moderate strength and, therefore, leads to a slight polarization. For extreme cases of the strong polarization, we examine two other sets, though they fit the spectra poorer. The SKSH1 set [16] includes no ΛNN force, so this interaction is core-contracting. We try also the 3rd set from [13] (hereafter R3) with a huge ΛNN force, which exemplifies a core-diluting interaction. In Table 1 the parameters for the Skyrme ΛN potential are presented.

All of the interactions employed do not involve a spin-orbit ΛN interaction. Note also that the standard scheme used [13] takes into account the ΛN spin-spin interaction in some effective averaging way, but the spin doublets remain unsplitted. Actually, we deal with the energies averaged over spin doublets.

Table 1. Parameters of Skyrme ΛN potentials

	λ_0 (MeV fm^3)	λ_1 (MeV fm^5)	λ_2 (MeV fm^5)	λ_3 (MeV fm^6)	Ref.
SKSH1	−176.5	−35.8	44.1	0.	[16]
YBZ5	−298.12	23.14	−23.14	2000.	[15]
R3	−628.5	179.5	−43.2	7539.	[13]

As the NN potential, the famous Sk3 set [21] is mainly used. Since this potential offers a great nuclear matter incompressibility (355 MeV), we examine also the softer SkM* set [22] with incompressibility 217 MeV.

2.2 $^{12}_{\Lambda}$Be and $^{16}_{\Lambda}$C

First, we discuss the model for the host nuclei ^{11}Be and ^{15}C. The ground ($1/2^+$) and the excited ($1/2^-$) bound states of ^{11}Be are treated as pure neutron $2s_{1/2}$ and $1p_{1/2}$ states, correspondingly, near the core ^{10}Be(0^+). Similarly, the ground ($1/2^+$) and the excited ($5/2^+$) states of ^{15}C are considered as $2s_{1/2}$ and $1d_{5/2}$ neutron states. For simplicity the possible configuration mixing is ignored. According to [7], we renormalize the central single-particle potential acting on the last neutron with factor k fitted to the experimental neutron separation energy. So, this potential becomes rather phenomenological but its shape is Hartree-Fock motivated. In hypernuclear calculations, we hold the same k for the nucleon-induced part of the neutron potential and do not renormalize the hyperon-induced part. Factors k and the rms radii of ^{11}Be and ^{15}C as well as the radii of the core nuclei ^{10}Be and ^{14}C are shown in Table 2. There exist considerable disagreements between the radii reported by various experimental groups. Moreover the adopted procedures of deducing of the radii from data are not perfect (see [26]). In this view, the calculated radii are compatible with empirical ones reasonably.

The renormalization procedure violates the self-consistency of the Hartree-Fock scheme. The total binding energies are almost unaffected by the renormalization. In this view, the neutron separation energies S_n are identified here with the calculated single-particle neutron energies e_n with opposite sign (i.e., $S_n = -e_n$) and not with the differences of the total binding energies of the related nuclei $B(^AZ) - B(^{A-1}Z)$ or, correspondingly, hypernuclei $B(^{A+1}_{\Lambda}Z) - B(^{A}_{\Lambda}Z)$. It means that we ignore the contribution of the core polarization by outer neutrons to the separation energy. In our calculations, the change of the core radii by the neutrons is about one percent that is less than the possible change by the hyperon. It justifies to some extent the approximation used. Moreover, as we consider mainly modifications of outer neutron states by hyperon, the core polarization by the neutrons, existing both in nuclei and hypernuclei, cannot affect the results crucially. However, related uncertainties should be

Table 2. Properties of neutron-rich nuclei and cores. The experimental values of the neutron separation energies (S_n) and the rms radii ($\langle r^2\rangle^{1/2}$, exp.) are listed. Calculated rms radii ($\langle r^2\rangle^{1/2}$, calc.) are obtained with factors k in the single particle potential fitted to experimental S_n

	neutron state	S_n (MeV)	$\langle r^2\rangle^{1/2}$, exp. (fm)	k Sk3	k SkM*	$\langle r^2\rangle^{1/2}$, calc. (fm) Sk3	$\langle r^2\rangle^{1/2}$, calc. (fm) SkM*
$^{10}\mathrm{Be}_{\mathrm{g.s.}}(0^+)$			$2.28 \pm 0.02^{a)}$ $2.479 \pm 0.028^{b)}$			2.35	2.39
$^{11}\mathrm{Be}_{\mathrm{g.s.}}(1/2^+)$	$2s_{1/2}$	0.505	$2.71 \pm 0.05^{a)}$ $3.039 \pm 0.038^{b)}$	1.19	1.08	2.99	3.07
$^{11}\mathrm{Be}^*(1/2^-)$	$1p_{1/2}$	0.180		0.80	0.76	2.89	2.99
$^{14}\mathrm{C}_{\mathrm{g.s.}}(0^+)$			$2.619 \pm 0.057^{b)}$			2.54	
$^{15}\mathrm{C}_{\mathrm{g.s.}}(1/2^+)$	$2s_{1/2}$	1.218	$2.783 \pm 0.092^{b)}$ $2.40 \pm 0.05^{c)}$	0.96		2.81	
$^{15}\mathrm{C}^*(5/2^+)$	$1d_{5/2}$	0.479		0.85		2.67	

$^{a)}$ [23], $^{b)}$ [24], $^{c)}$ [25]

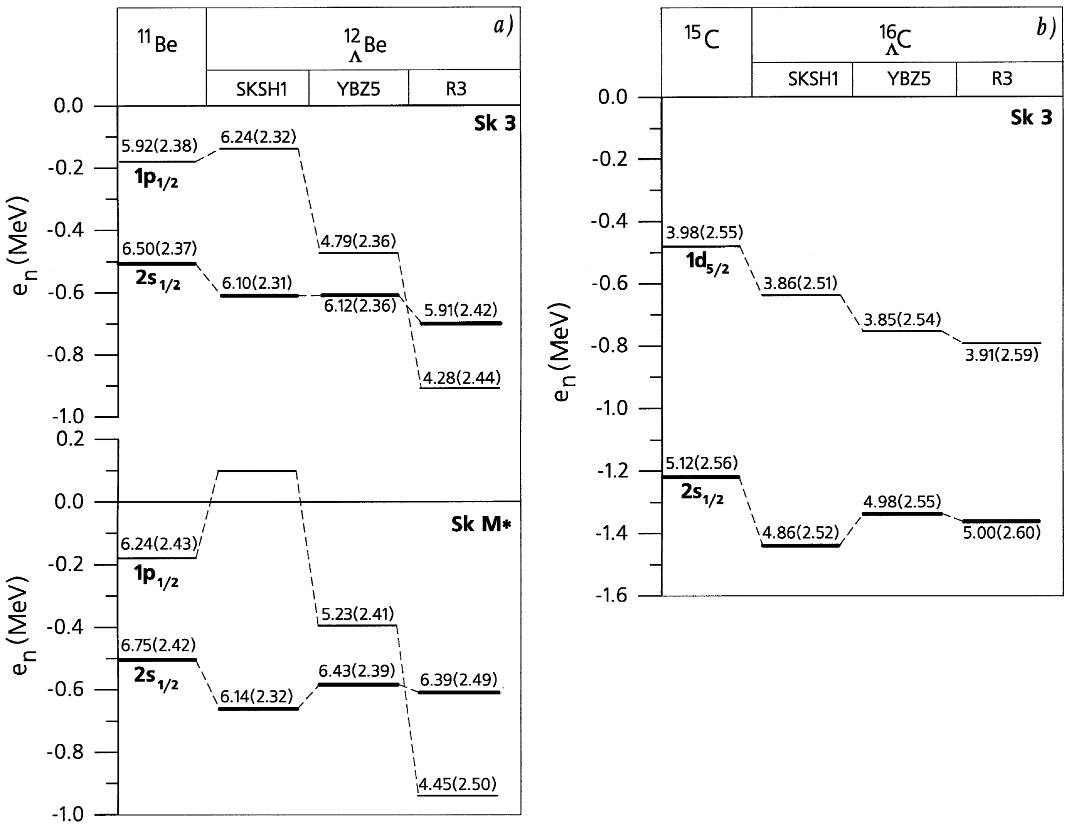

Fig. 1. External neutron single-particle energies e_n (MeV) and corresponding rms radii (fm) of the last neutron and the core (in parentheses) for $^{12}_{\Lambda}$Be (a) and $^{16}_{\Lambda}$C (b). ΛN interactions are SKSH1, YBZ5, and R3. NN interactions are Sk3 and SkM*. Thick lines are for $2s$ states, and thin ones are for $1p$ and $1d$ states

stressed. The role of the core polarization by the outer neutrons deserves further study.

The neutron single-particle energies e_n as well as rms radii of the nuclear core and the last neutron orbit are presented in Fig. 1 for $^{12}_{\Lambda}$Be and $^{16}_{\Lambda}$C and various ΛN and NN interactions.

It is seen that the $1p_{1/2}$ halo state energy in $^{12}_{\Lambda}$Be is quite sensitive to the ΛN interaction. The most unexpected result is obtained with the core-contracting SKSH1 set: hyperon addition pushes the halo state upward to the threshold despite the hyperon attraction. Moreover, if the nuclear incompressibility is low (the SkM* set), this state becomes unbound. Otherwise, the core-diluting R3 set predicts a rather tightly bound $1p_{1/2}$ state which becomes the ground state of $^{12}_{\Lambda}$Be. The nonpolarizing YBZ5 set represents an intermediate case. Evidently, the neutron

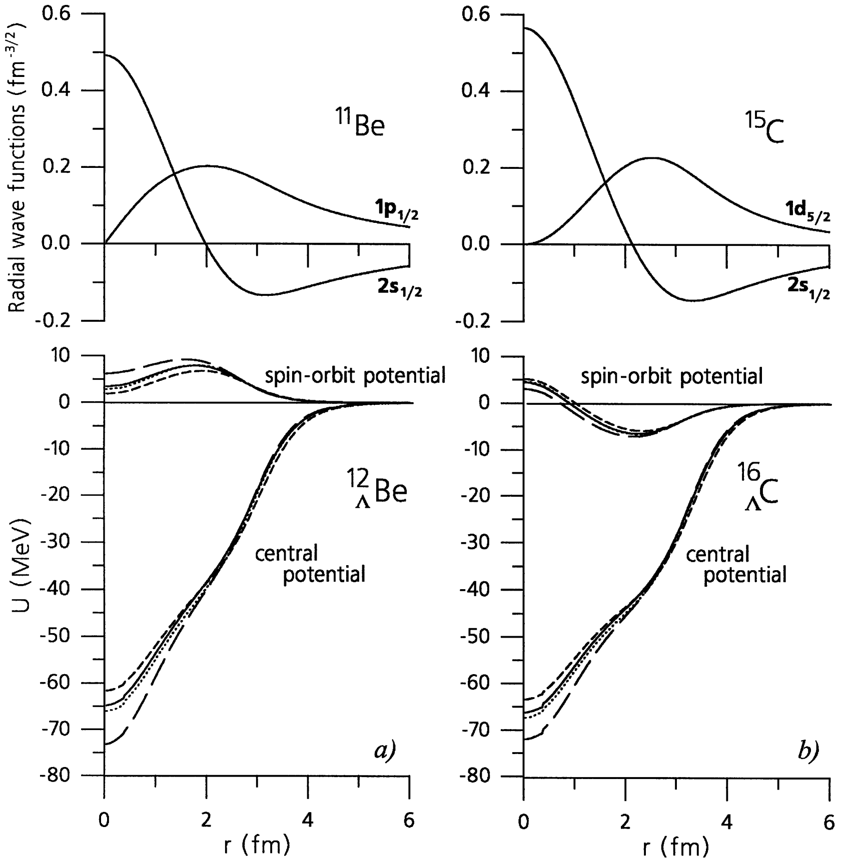

Fig. 2. Lower part: a) Single-particle potentials acting on the halo neutron in ^{11}Be (solid line) and in $^{12}_{\Lambda}$Be with the ΛN potentials SKSH1 (long-dashed), R3 (short-dashed), and YBZ5 (dotted). NN potential is Sk3. b) The same for ^{15}C and $^{16}_{\Lambda}$C. Upper part: Single-neutron wave functions in various states of ^{11}Be (a) and ^{15}C (b)

separation energy is strongly correlated with the polarizing property of the ΛN interaction.

One can see that the $1d_{5/2}$ state in $^{16}_{\Lambda}$C is far less sensitive to the ΛN interaction: in all the cases the last neutron becomes more bound. The difference between e_n for the various ΛN potentials is not greater than 0.2 MeV whereas the same quantity for $^{12}_{\Lambda}$Be is as much as 1 MeV.

The $2s$ neutron states do not reveal a meaningful response to the hyperon addition.

It is noteworthy that the possible decrease of the neutron separation energy ($S_n(^{12}_{\Lambda}\text{Be}) < S_n(^{11}\text{Be})$ with the SKSH1 set) means that B_Λ can decrease with A, as seen evidently from the relation: $S_n(^{A+1}_{\Lambda}Z) = S_n(^{A}Z) + B_\Lambda(^{A+1}_{\Lambda}Z) - B_\Lambda(^{A}_{\Lambda}Z)$. It seems rather strange since B_Λ is believed to grow with A. We recall this point in the next section.

The valence neutron orbit radii (indicated in Fig. 1) are usually in natural connection with the separation energies. They become greater when S_n is reduced, and decrease with S_n increasing. The values of the core radii (shown in parentheses) reflect the properties of ΛN interaction used. The radii become smaller for core-contracting SKSH1, and rise for R3, and they change by few percent at most.

To elucidate the interplay between the core polarization and the single-neutron state properties, we compare the single-particle potentials acting on the last neutron in the nucleus and in the related hypernucleus. The central and spin-orbit (SO) potentials are shown in Fig. 2 (lower part) for ^{11}Be and $^{12}_{\Lambda}$Be, ^{15}C and $^{16}_{\Lambda}$C. The polarization implication is twofold. For instance, let us consider the core contraction. Firstly, the core density distribution and, therefore, the central potential become slightly narrower. Since the last neutron moves in p and d states outside the central region (see the upper part of Fig. 2), it responds to this very small reduction of the potential range rather than to the additional hyperon attraction in the center. So the central potential becomes less attractive effectively. On the other hand, the SO potential is linear in the derivative of the core density, so it is enhanced due to the contraction since the core edge is sharpened. The central and SO potential modifications combine constructively in $1p_{1/2}$ states (the growing SO potential is repulsive) and destructively in $1d_{5/2}$ states (the SO potential is attractive). Thus, the $1p_{1/2}$ state becomes less bound despite hyperon attraction, and the position of the $1d_{5/2}$ level is fairly stable.

If the core dilutes, the related mechanism is similar but opposite. The central potential expands, whereas the SO one reduces. This leads to substantial additional attraction for the $1p_{1/2}$ state.

The SO neutron potential is naturally of particular significance for the weakly bound neutrons due to its periphericity. The importance of the SO nucleon potential in halo hypernuclei has been emphasized in [12]. Another aspect of SO potential implication in properties of hypernuclei with neutron excess was inferred from the relativistic mean field framework by Vretenar *et al.* [27], who pointed out the crucial role of the SO interaction in stabilizing of an extremely neutron-rich core.

Such sensitivity of halo hypernuclear properties to nucleonic SO interaction can be useful in view of currently discussed problem of possible modification of SO force in nuclear neutron-rich systems (e.g., [28]).

It is seen also from Fig. 2 that the $2s$ wave function has a node just in the region of central potential edge. Additionally, $2s$ neutrons evidently move in the central region for some time and feel the Λ central attraction. Therefore, the reduction of the central potential range is insignificant in $2s$ states.

It is noteworthy that the level ordering in halo systems appears to be very sensitive to the shape of the core surface. Even a very slight shallowing of the surface can convert the abnormal ordering to the normal one as for the R3 potential in $^{12}_{\Lambda}$Be.

2.3 $^{11}_{\Lambda}$Li

Several experiments have been devoted to the ^{10}Li spectrum ([29–31], see also [32] and references therein), but the situation is still unclear. This nucleus is unbound, and the interpretation of the existing data is ambiguous. Moreover, unlike $^{12}_{\Lambda}$Be and $^{16}_{\Lambda}$C, the core ^{9}Li has a nonzero spin, so each single-particle neutron state ($2s_{1/2}$ or $1p_{1/2}$) forms a doublet in the ^{10}Li spectrum. Probably, the ^{10}Li ground state is constructed by the $2s_{1/2}$ neutron and lies just closely to the threshold and the $1p$ states are somewhat higher [30]. In view of the uncertainties, we didn't try to make quantitative predictions for $^{11}_{\Lambda}$Li, but only checked the hyperon influence on the single-particle energy of the last neutron. In our calculation we consider the ground state as the $2s$ neutron state and fit its single-particle energy to zero for specificity. The related renormalization factor with the Sk3 set is $k = 1.28$. For the excited state with $1p_{1/2}$ neutron we choose for example $S_n = -0.42$ MeV ($k = 0.87$), which appears to be the average of the spin doublet [30,31].

Though the energy of the $2s$ state in ^{10}Li is probably about 0.05 MeV, the shift due to hyperon in our calculations is quite small ($0.02-0.03$ MeV at most) regardless of the interaction used. So, this state likely remains unbound. The $1p_{1/2}$ state can be lowered much more considerably (by about 0.3 MeV) for YBZ5 set, but remains unbound too for the adopted $S_n = -0.42$ MeV value. However, possibly one of the spin doublet members in ^{9}Li+$n(1p_{1/2})$ lies at approximately $S_n = -0.2$ MeV [29,30]. Therefore this state can turn to be bound in $^{11}_{\Lambda}$Li. Alternatively, the $1p$ state can be bound if the ΛN interaction is slightly more diluting than the YBZ5 set (for the extremely diluting R3 set we obtain the shift of the $1p$ neutron state as much as 0.9 MeV, similar to that in $^{12}_{\Lambda}$Be).

Therefore, while the $2s$ state likely cannot be bound by hyperon, the $1p$ one possibly can. In other words, if bound $^{11}_{\Lambda}$Li exists then it is most likely the $1p$ halo state, which energy is sensitive to ΛN interaction features.

Helium isotopes $^{11}_{\Lambda}$He and $^{9}_{\Lambda}$He have been briefly considered in [12]. Similarly to the considerations above, bound $^{11}_{\Lambda}$He is possible if the ΛNN force is sufficiently strong.

3 Dependence of Λ binding energies on Z at fixed A

The neutron separation energy S_n and the Λ binding energy B_Λ are both defined by the total binding energy $B(^{A+1}_{\Lambda}Z)$. However, evaluation of the former requires also $B(^{A}_{\Lambda}Z)$, none of those is known now for the species considered above. On the other hand, evaluation of B_Λ incorporates well known nuclear total binding energies $B(^{A}Z)$.

To avoid ambiguities in the calculated total energies originating from the renormalization scheme used above, we study here Z dependence of B_Λ at fixed A in the consistent Hartree-Fock approach for hypernuclei without prominent halos. Whereas A dependence of B_Λ has been studied in details by many authors, Z dependence is investigated much poorer. We also overview briefly available data in this line.

In Table 3, the calculated binding energies for A=12 and 16 and various $Z \leq N$ are shown. It is seen that the dependence of B_Λ in $^{13}_{\Lambda}Z$ can be strong, and B_Λ behavior is determined by the strength of the ΛNN force. B_Λ falls rapidly in neutron-rich systems at zero ΛNN force, and, otherwise, grows when the ΛNN force is great. In fact, B_Λ depends on nuclear density in central region. It has been shown [20] that the Λ binding energy increases with the nuclear central density for a small ΛNN force, and, otherwise, the higher is the central density the smaller is B_Λ when the three-body force is great. As ^{12}C nuclear density is clearly most centered among the nuclei of A=12 (Fig. 3), B_Λ falls with the neutron excess increase for the pure two-body ΛN potential (SKSH1), and grows when the ΛNN interaction is strong (R3).

On the other hand, the relationship between the strength of three-body force and the Z dependence of B_Λ is reverse (and more slight) in $^{17}_{\Lambda}Z$. Our calculations show that the r=0 density at A=16 is maximal in the asymmetric nucleus ^{16}C (Fig. 3), but the densities of ^{16}O, ^{16}N, and ^{16}C cross over not far from the center (at $r \approx 1.7-1.8$ fm), so their difference in the whole central region is not prominent. The central densities are effectively close to each other with Sk3 set, whereas the SkM* set gives somewhat more dense center of ^{16}C. This results in reverse B_Λ dependencies on Z at A=16 in the latter case with respect to those at A=12.

Note that the usual shell model ordering is chosen here, so the ground state of ^{12}Be includes the closed $1p$ neutron shell whereas that of ^{16}C includes the $(1d_{5/2})^2$ neutron pair. As the actual ^{12}Be and ^{16}C offer, probably, substantial $(2s_{1/2})^2$ configurations, the central densities may be higher, and the effect may be greater in $^{17}_{\Lambda}$C and less in $^{13}_{\Lambda}$Be.

It is seen that B_Λ can fall substantially when host nucleus becomes looser, if the ΛNN force is weak. It justifies possible decrease of B_Λ in $^{12}_{\Lambda}$Be with respect to a more tight $^{11}_{\Lambda}$Be pointed in Sec. 2.

Table 3. Binding energies B_Λ (MeV) in ${}^{13}_{\Lambda}Z$ and ${}^{17}_{\Lambda}Z$ for different NN and ΛN potentials

NN potential	ΛN potential	${}^{13}_{\Lambda}$Be	${}^{13}_{\Lambda}$B	${}^{13}_{\Lambda}$C	${}^{17}_{\Lambda}$C	${}^{17}_{\Lambda}$N	${}^{17}_{\Lambda}$O
	SKSH1	10.7	11.6	12.4	13.3	13.2	13.1
Sk3	YBZ5	11.1	11.3	11.2	13.4	13.4	13.5
	R3	13.2	12.1	11.4	14.0	14.1	14.3
	SKSH1	10.3	11.9	13.3	13.7	13.4	13.1
SkM*	YBZ5	10.7	11.1	11.2	13.2	13.2	13.3
	R3	13.9	12.6	11.4	14.3	14.5	14.9

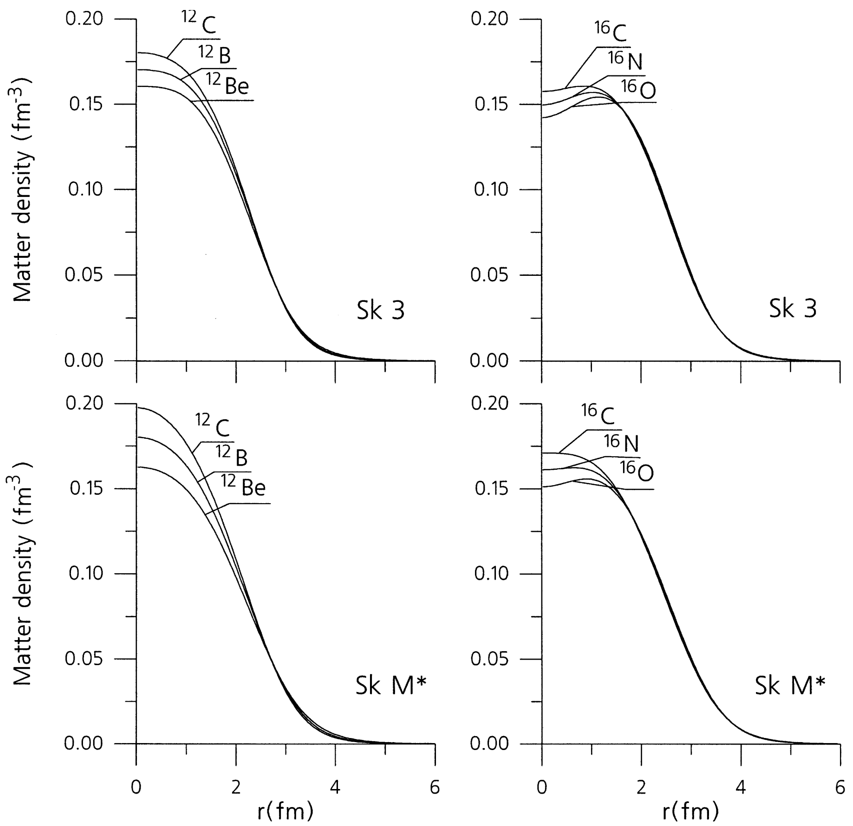

Fig. 3. Density distributions in nuclei ${}^{12}Z$ (left) and ${}^{16}Z$ (right) with the Sk3 and SkM* potentials

Some data on B_Λ's in hypernuclei with different Z at the same A are already known from emulsion experiments ([33] and refs. therein). Differences in B_Λ's can result not only from variations of nuclear densities as described above, but also from other factors such as the charge symmetry breaking (CSB) ΛN interaction and the Coulomb energies. It may be suggested that the variations of the densities are smallest in analog-core hypernuclei, so these systems are most suitable for extraction of the CSB interaction. A comprehensive analysis of these problem for ${}^{4}_{\Lambda}$H ($B_\Lambda = 2.04 \pm 0.04$ MeV) and ${}^{4}_{\Lambda}$He ($B_\Lambda = 2.39 \pm 0.03$ MeV) has been made by Bodmer and Usmani [34]. These data imply that the Λp attraction is stronger than the Λn one. However, another known pair, ${}^{12}_{\Lambda}$B ($B_\Lambda = 11.37 \pm 0.06$ MeV) and ${}^{12}_{\Lambda}$C ($B_\Lambda = 10.76 \pm 0.19$ MeV), indicates the opposite sign of CSB. The difference in the last B_Λ's was supposedly attributed to the core contraction and the subsequent rising of the Coulomb energies [35]. However, our Skyrme-Hartree-Fock calculation even for the strongly contracting charge-symmetrical SKSH1 set with different NN interactions gives for $B_\Lambda({}^{12}_{\Lambda}\mathrm{B}) - B_\Lambda({}^{12}_{\Lambda}\mathrm{C})$ values not greater than 0.2 MeV, much lower than the experimental difference. Thus, this difference unlikely can be explained by the Coulomb energies only. There exist also several analog-core hypernuclei with nearly equal B_Λ [33,35]. So the CSB problem is rather unclear now. That is why we did not incorporate the CSB ΛN interaction in our calculation.

Variations of core central densities can evidently be exhibited in nonanalog-core hypernuclei at the same A. The following data are known [33]: $B_\Lambda({}^{7}_{\Lambda}\mathrm{Li}) = 5.58 \pm 0.03$ MeV versus $B_\Lambda({}^{7}_{\Lambda}\mathrm{Be}) = 5.16 \pm 0.08$ MeV and $B_\Lambda({}^{9}_{\Lambda}\mathrm{Be}) = 6.71 \pm 0.04$ MeV versus $B_\Lambda({}^{9}_{\Lambda}\mathrm{Li}) = 8.50 \pm 0.12$ MeV as well as $B_\Lambda({}^{9}_{\Lambda}\mathrm{B}) = 8.29 \pm 0.04$ MeV. Unfortunately, all these species are beyond the applicability of our approach, so they may be discussed only qualitatively here. It is natural to suggest that the $^7_\Lambda$Li core is more centered than the $^7_\Lambda$Be one. Then it may be suggested that the interaction needed incorporates a low ΛNN component. It is compatible with the recent calculation by Hiyama *et al.* [36], who obtained $B_\Lambda(^7_\Lambda\mathrm{Li}) > B_\Lambda(^7_\Lambda\mathrm{Be})$ and the core radius of $^7_\Lambda$Li smaller than that of $^7_\Lambda$Be in a microscopic calculation with a purely attractive ΛN potential[1]. It is difficult to calculate the densities of the A=8 cores in a unified way, but it is reasonable to suggest that the small $B_\Lambda(^9_\Lambda\mathrm{Be})$ results also from a minimum of the core density at the center, that indicates again to a low three-body force (a contracting ΛN interaction). However, more data are needed for definite conclusions. In this view, the $(e, e'K^+)$ and (K^-, π^0) reactions at the targets already studied in the (π^+, K^+) or (K^-, π^-) reactions are quite promising.

Generally, we show that the dependence of B_Λ on neutron excess at fixed A, firstly, is sensitive to the ΛN interaction features, and, secondly, is intimately related with the central nuclear densities.

4 Outlook

We calculated properties of Λ hypernuclei with neutron excess and showed these properties to be sensitive remarkably to ΛN and NN interaction features. Their interplay is partly of nontrivial and unexpected essence, particularly, the dependence on the ΛNN force, on the neutron-core spin-orbit potential, and on the nuclear incompressibility. Ambiguities in both the ΛN and NN interactions render impossible quantitative predictions of the properties of neutron-rich hypernuclei. Nevertheless, it is seen that such systems can be considered as a source of information complementary to that from ordinary hypernuclei. Also some other properties of hypernuclear interaction that were not taken into account here, may be the subject of further study regarding to these hypernuclei.

Firstly, it should be noted that the spin-spin ΛN interaction leads to doublet splitting, i.e., all the hypernuclear states in $^{12}_\Lambda$Be, $^{11}_\Lambda$Li, and $^{16}_\Lambda$C considered above are really pairs of levels. The magnitude of the splitting is known reliably only for $^4_\Lambda$H, $^4_\Lambda$He, and $^7_\Lambda$Li. Various models give different magnitudes of the splitting and even the different level ordering in various hypernuclei [38–40]. It may be expected that the spin-spin interaction between the hyperon and the outer neutron in $^{12}_\Lambda$Be and $^{16}_\Lambda$C is small due to a small overlap of their spatial distributions. On the other hand, the spin-spin ΛN interaction involves the core nucleons in $^{11}_\Lambda$Li and may be greater.

[1] Very recently, a direct estimation of the core radius in $^7_\Lambda$Li from the $B(E2)$ measurement were reported [10], which confirmed the core contraction. The core contraction in $^7_\Lambda$Li was also deduced from a shell model analysis of the magnitude of the spin doublet splitting [37].

Another point ignored in this paper is the charge symmetry breaking ΛN interaction. We did not incorporate this interaction in our calculation since even the sign of the related effect is doubtful now.

Lastly, the neutron separation energies can be also influenced by the spin-orbit ΛN interaction which strength was preliminary argued from the recent KEK experiment [41] and the reanalysis of the old emulsion data [42].

Summing up, neutron-rich Λ hypernuclei may be considered to be of twofold interest. Measurements of their properties can give an information both on the halo system dynamics and on the hypernuclear interactions especially if various hypernuclei of such type will be observed. For quantitative predictions, more consistent nuclear models as well as extended knowledge of the ΛN interaction are needed.

Another important question is about possibility of production of such hypernuclei. Firstly, in the recent (K^-, π^+) experiment with stopped kaons, upper limits for the production rates were established, which are as small as $6 \cdot 10^{-5}$ for $^{12}_\Lambda$Be, $^{16}_\Lambda$C and $2 \cdot 10^{-4}$ for $^9_\Lambda$He per a stopped kaon [3]. As for in-flight experiments in the kinematic region corresponding to the Λ production, only inclusive reaction $^3\mathrm{He}(K^-, \pi^+)$ has been studied experimentally [43]. The cross section is by two orders of magnitude smaller than that for the one step (K^-, π^-) reaction. Such two-step process can be realized through the Λ production in the $(\bar{K}, \pi)$ strangeness exchange with later (preceding) $\pi(\bar{K})$ charge exchange [44] as well as through virtual Σ^- production with subsequent $\Sigma^- p \to \Lambda n$ transition [45]. Optimal kinematic conditions should be searched for. Also the similar reaction (π^-, K^+) may be considered. On the other hand, an alternative way for production of the neutron-rich hypernuclei may be heavy ion collisions. Some estimations for the related cross sections have been done in [46], and the cross sections for production of, e.g., $^{12}_\Lambda$Be and $^{16}_\Lambda$C, appear to be not extremely small, though further theoretical study is, of course, needed. This way is more universal and feasible for hypernuclei not attainable in the (K^-, π^+) and (π^-, K^+) reactions, as, e.g., $^{11}_\Lambda$He.

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